Guided Notes

MAC 2312
Section 7.2
Trig Integrals

Basic strategy — power of cosine odd

$$\int \sin^m x \cos^{2k+1} x\,dx = \int \sin^m x\,(\cos^2 x)^k \cos x\,dx$$
$$= \int \sin^m x\,(1-\sin^2 x)^k \cos x\,dx$$

then let $u = \sin x$

Basic Strategy — power of sine is odd

$$\int \sin^{2k+1} x \cos^n x\,dx = \int (\sin^2 x)^k \cos^n x \sin x\,dx$$
$$= \int (1-\cos^2 x)^k \cos^n x \sin x\,dx$$

then let $u = \cos x$

If both are odd, you may use either.

Basic Strategy — both sin & cos are even

use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin 2x = 2\sin x \cos x$

7.2 Examples

(2) $\int \sin^3\theta \cos^4\theta \, d\theta$

$= \int \cos^4\theta \sin^2\theta \sin\theta \, d\theta$

$= \int \cos^4\theta (1-\cos^2\theta) \sin\theta \, d\theta$

$= \int \cos^4\theta \sin\theta \, d\theta - \int \cos^6\theta \sin\theta \, d\theta$

$u=\cos\theta$ $\qquad\qquad$ $u=\cos\theta$

$du=-\sin\theta \, d\theta$ $\qquad\qquad$ $du=-\sin\theta \, d\theta$

$= \boxed{\frac{-\cos^5\theta}{5} + \frac{\cos^7\theta}{7} + C}$

(10) $\int_0^{\pi} \sin^2 t \cos^4 t \, dt$

$= \frac{1}{4}\int_0^{\pi} 4\sin^2 t \cos^2 t \cos^2 t \, dt$

$= \frac{1}{4}\int_0^{\pi} (2\sin t \cos t)^2 \cos^2 t \, dt$

$= \frac{1}{4}\int_0^{\pi} (\sin 2t)^2 \left(\frac{1}{2}(1+\cos 2t)\right) dt$

$= \frac{1}{4}\cdot\frac{1}{2}\int_0^{\pi} \left((\sin 2t)^2 + \underbrace{(\sin 2t)^2 \cos 2t}_{\text{u sub}}\right) dt$

use $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $\qquad\qquad$ $u=\sin 2t$

(10) continued

$$= \frac{1}{8}\int_0^{\pi} \tfrac{1}{2}(1-\cos 4t)\,dt + \frac{1}{8}\int_0^{\pi} (\sin 2t)^2 \cos 2t\,dt$$

$$= \frac{1}{8}\cdot\frac{1}{2}\left[t - \frac{1}{4}\sin 4t \,\right|_0^{\pi} + \frac{1}{8}\left[\frac{1}{2}\frac{(\sin 2t)^3}{3}\,\right|_0^{\pi}$$

$$= \frac{1}{16}\left[\left(\pi - \frac{1}{4}\cancelto{0}{\sin(4\pi)}\right) - \left(0 - \frac{1}{4}\cancelto{0}{\sin(0)}\right)\right]$$

$$+ \frac{1}{8}\left[\frac{1}{2}\frac{\cancelto{0}{(\sin 2\pi)^3}}{3} - \frac{1}{2}\frac{\cancelto{0}{(\sin 0)^3}}{3}\right] = \boxed{\frac{\pi}{16}}$$

(20) $\boxed{\int x \sin^3 x\,dx} = \int x \cdot \sin^2 x \cdot \sin x\,dx$

$= \int x(\sin^3 x)\,dx$ use Integration by parts

D	I
x	$\sin^3 x$
1	

$\int \sin^3 x\,dx$

$= \int \sin x(\sin^2 x)\,dx$

$= \int \sin x\,(1-\cos^2 x)\,dx$

$= \int \sin x\,dx - \int \cos^2 x \sin x\,dx$ $\quad u = \cos x$, $\frac{du}{dx} = -\sin x$

$= -\cos x + \dfrac{\cos^3 x}{3}$

(continued)

(20) By parts continued

	D	I
$+$	x	$\sin^3 x$
$-$	1	$-\cos x + \dfrac{\cos^3 x}{3}$
$+$	0	$\underbrace{-\sin x + \frac{1}{3}\sin x}_{-\frac{2}{3}\sin x} - \dfrac{(\sin x)^3}{9}$

$$\frac{1}{3}\int \cos^3 x\,dx = \frac{1}{3}\int \cos x\,(\cos^2 x)\,dx$$

$$= \frac{1}{3}\int \cos x\,(1 - \sin^2 x)\,dx$$

$$= \frac{1}{3}\int (\cos x - \sin^2 x \cdot \cos x)\,dx$$

$$= \frac{1}{3}\int \cos x\,dx - \int (\sin x)^2 \cos x\,dx$$

$$= \frac{1}{3}\sin x - \frac{(\sin x)^3}{3} \cdot \frac{1}{3}$$

$u = \sin x$
$du = \cos x\,dx$

$$= \boxed{-x\cos x + \frac{x\cos^3 x}{3} + \frac{2}{3}\sin x + \frac{1}{9}(\sin x)^3 + c}$$

$\boxed{\text{Strategy for tangent and secant}}$

$\boxed{\text{even secant}}$

$$\int \tan^m x \sec^{2k} x \, dx$$

$$= \int \tan^m x \, (\sec^2 x)^{k-1} \sec^2 x \, dx$$

$$= \int \tan^m x \, (1 + \tan^2 x)^{k-1} \sec^2 x \, dx$$

$\boxed{\text{let } u = \tan x}$

$\boxed{\text{Odd tangent}}$ use $\tan^2 x = \sec^2 x - 1$

$$\int \tan^{2k+1} x \sec^n x \, dx$$

$$= \int (\tan^2 x)^k \sec^{n-1} x \sec x \tan x \, dx$$

$$= \int (\sec^2 x - 1)^k (\sec^{n-1} x) \sec x \tan x \, dx$$

$\boxed{u = \sec x}$

$\boxed{\text{Recall}}$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

24 $\int (\tan^2 x + \tan^4 x)\,dx$

$= \int \tan^2 x\,(1 + \tan^2 x)\,dx$

$= \int \tan^2 x\,(\sec^2 x)\,dx$

$= \int (\sec x \tan x)^2\,dx$ let $u = \tan x$, $du = \sec^2 x\,dx$

$= \int (\tan x)^2 \sec^2 x\,dx$

$= \boxed{\dfrac{\tan^3 x}{3} + c}$

34 $\int \dfrac{\sin\phi}{\cos^3\phi}\,d\phi = \int \cos^{-3}\phi\,\sin\phi\,d\phi$ $u = \cos\phi$, $du = -\sin\phi\,d\phi$

$= -\dfrac{\cos^{-2}\phi}{-2} + c = \boxed{\dfrac{1}{2\cos^2\phi} + c}$

$= \boxed{\frac{1}{2}\sec^2\phi + c}$

OR

$\int \dfrac{\sin\phi}{\cos\phi} \cdot \dfrac{1}{\cos^2\phi}\,d\phi = \int \tan\phi\,\sec^2\phi\,d\phi$ $u = \tan\phi$, $du = \sec^2\phi\,d\phi$

$= \boxed{\frac{1}{2}\tan^2\phi + c}$

$$\boxed{(38)\ \int_{\pi/4}^{\pi/2} \csc^4\theta \cot^4\theta \, d\theta}$$

$\cot\frac{\pi}{2} = 0$

$\cot\frac{\pi}{4} = 1$

$$= \int_{\pi/4}^{\pi/2} \cot^4\theta \csc^2\theta \csc^2\theta \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \cot^4\theta (1+\cot^2\theta) \csc^2\theta \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \cot^4\theta \csc^2\theta \, d\theta + \int_{\pi/4}^{\pi/2} \cot^6\theta \csc^2\theta \, d\theta$$

$u = \cot\theta$ $\qquad$ $u = \cot\theta$

$du = -\csc^2\theta \, d\theta$ $\qquad$ $du = -\csc^2\theta \, d\theta$

$$= -\frac{(\cot\theta)^5}{5}\Bigg|_{\pi/4}^{\pi/2} + -\frac{(\cot\theta)^7}{7}\Bigg|_{\pi/4}^{\pi/2}$$

$$= \frac{-1}{5}\left[\left(\overset{0}{\cancel{\cot\tfrac{\pi}{2}}}\right)^5 - \overset{1}{\cancel{\cot\left(\tfrac{\pi}{4}\right)}}^5\right] + \frac{-1}{7}\left[\left(\overset{0}{\cancel{\cot\tfrac{\pi}{2}}}\right)^7 - \overset{1}{\cancel{\cot\left(\tfrac{\pi}{4}\right)}}^7\right]$$

$$= \frac{-1}{5}\left[-(1)^5\right] + \frac{-1}{7}\left[-(1)^7\right]$$

$$= \frac{1}{5} + \frac{1}{7} = \boxed{\frac{12}{35}}$$

(48) $$\int \frac{dx}{\cos x - 1}$$

$$= \int \frac{1}{\cos x - 1} \cdot \frac{\cos x + 1}{\cos x + 1}\, dx$$

$$= \int \frac{\cos x + 1}{\cos^2 x - 1}\, dx$$

$$= \int \frac{\cos x + 1}{-\sin^2 x}\, dx$$

$$= \int \left(\frac{\cos x}{-\sin^2 x} - \frac{1}{\sin^2 x} \right) dx$$

$$= \int \left(\frac{\cos x}{-\sin x} \cdot \frac{1}{\sin x} - \frac{1}{\sin^2 x} \right) dx$$

$$= \int -\cot x \cdot \csc x - \csc^2 x)\, dx$$

$$= \boxed{\csc x + \cot x + C}$$